Composable and Finite Computational Security of Quantum Message Transmission

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ETH Zurich, Switzerland

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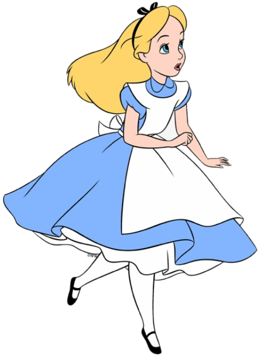

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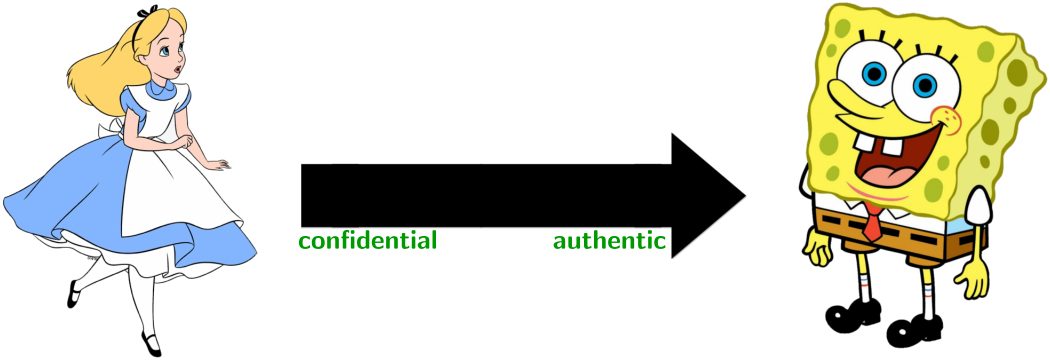

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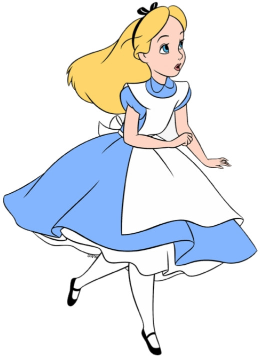

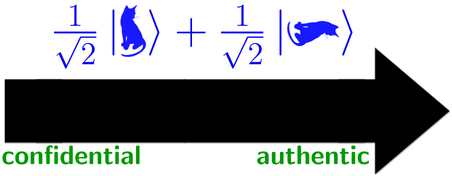

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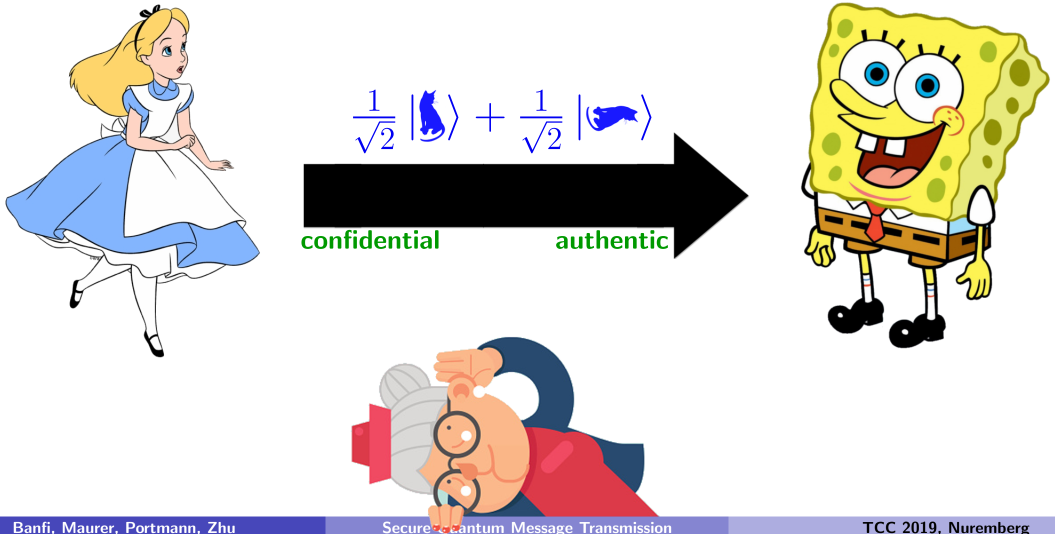

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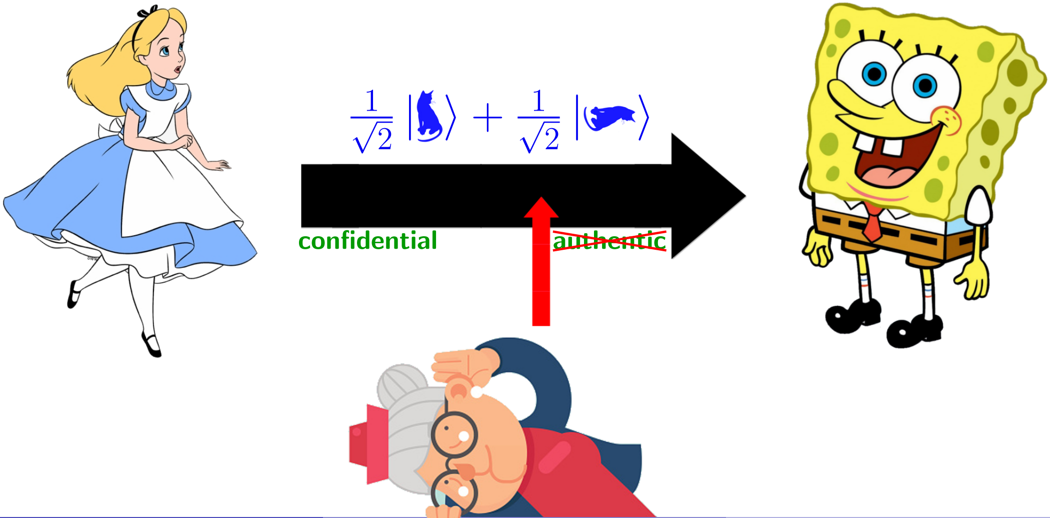

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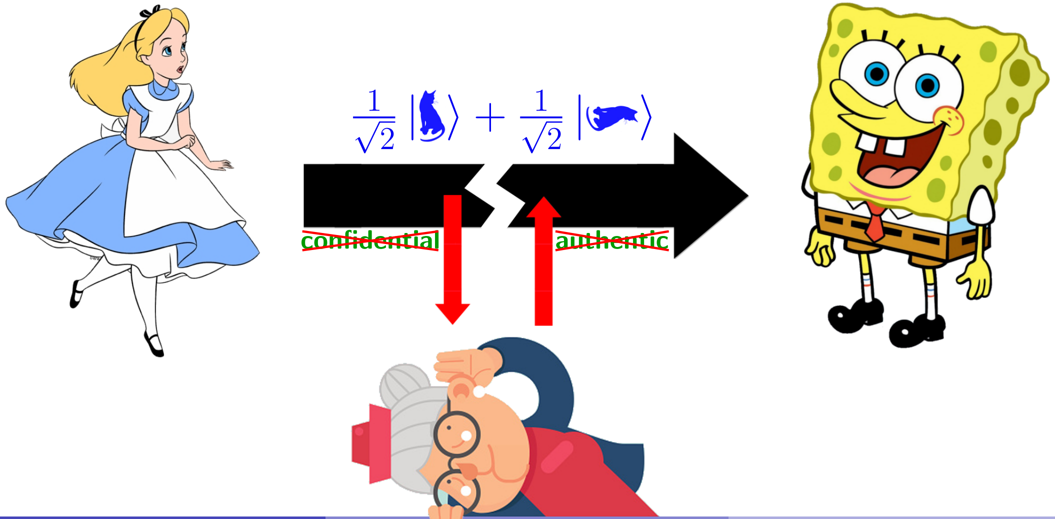

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Defining the **ideal** oracles (simplified for 1 message):

registers P, C

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    if [M̃_0, M_1] ≡ |Φ+⟩ then
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$$\begin{aligned}&\textbf{oracle } \mathbf{IdealDec}_K(\sigma)\\&\quad \textbf{register } \tilde{M}_0 \leftarrow \mathbf{Dec}_K(\sigma)\\&\quad \textbf{if } [\tilde{M}_0, M_1] \equiv |\mathbf{\Phi}^+\rangle \textbf{ then}\\&\qquad \textbf{return } P\\&\quad \textbf{return } |\bot\rangle\end{aligned}$$

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$\Longrightarrow$ It is possible to relate the two notions

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- **Finite statements:** concrete reductions to hardness assumptions
    - Crucial for real-world implementations, appreciated by the Experimental QCrypt community

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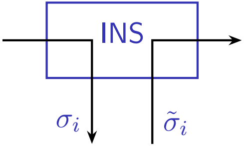

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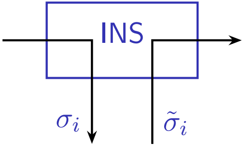

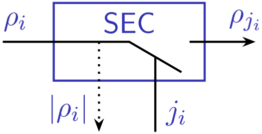

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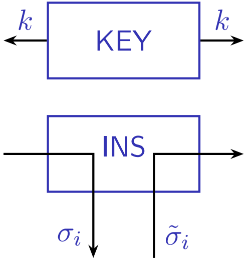

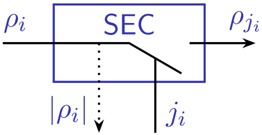

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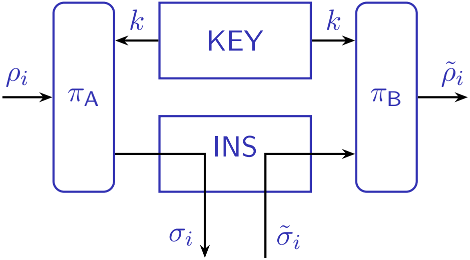

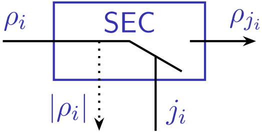

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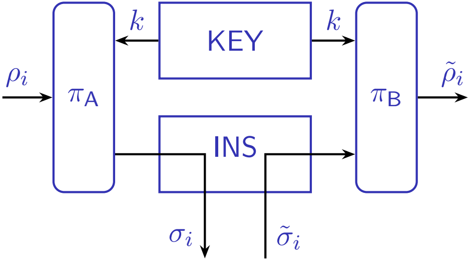

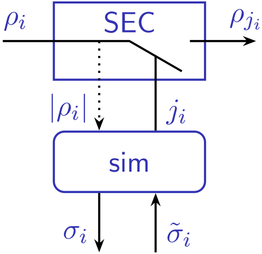

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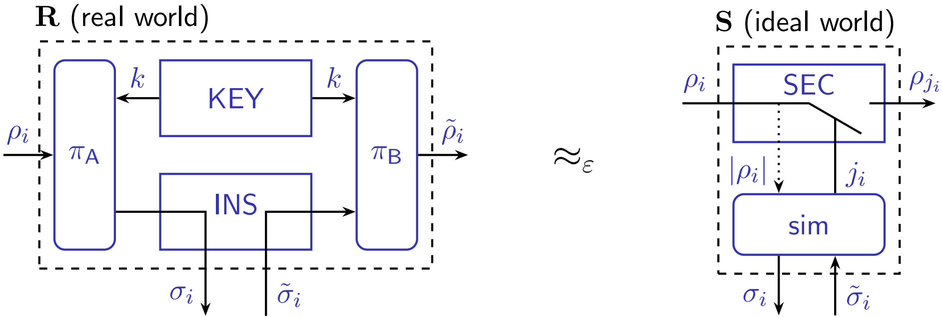

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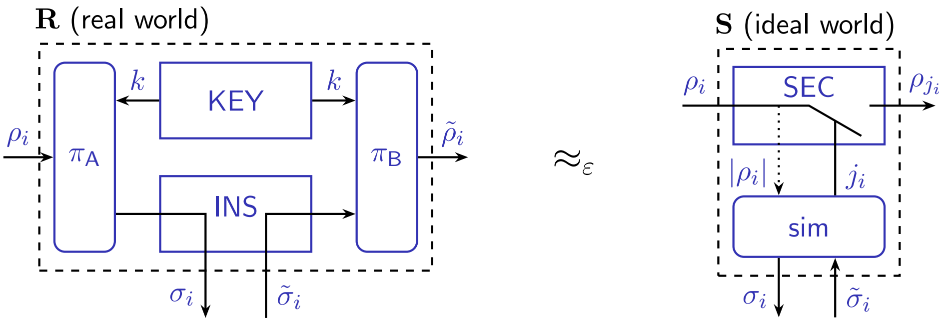

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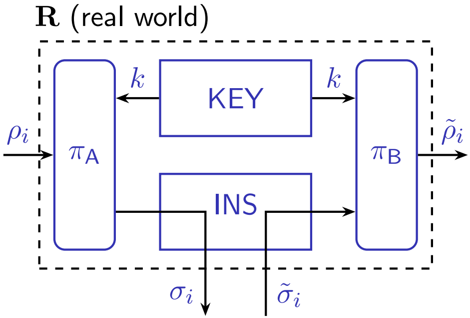
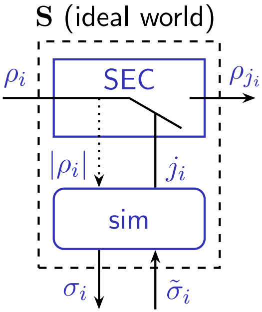


$\approx_\varepsilon$



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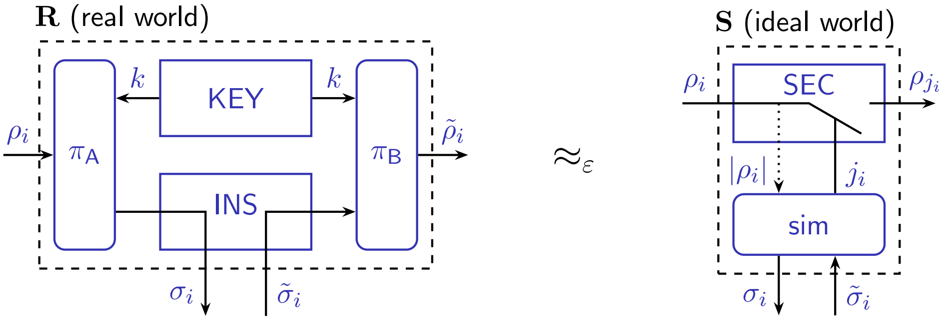

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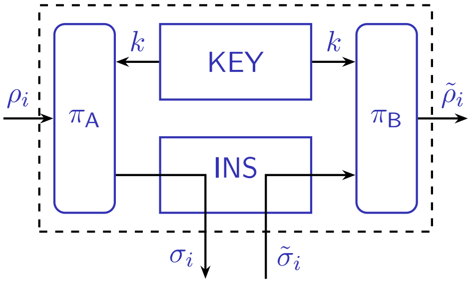

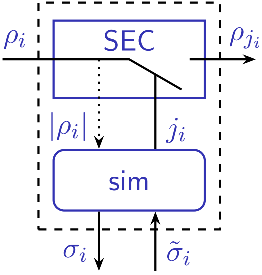

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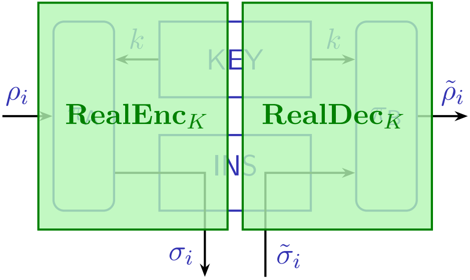

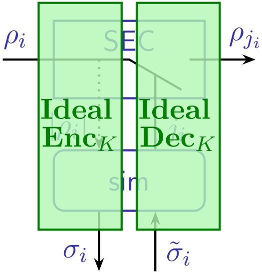

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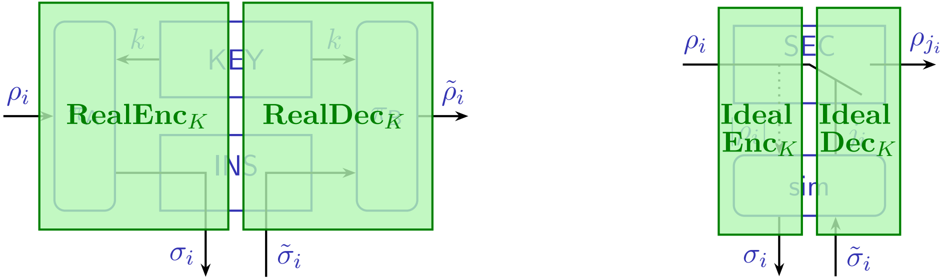

Theorem

QAE is *composable security (conf. + auth.)* with a simulator hard-coded.

Recall: composable security (= confidentiality + authenticity)

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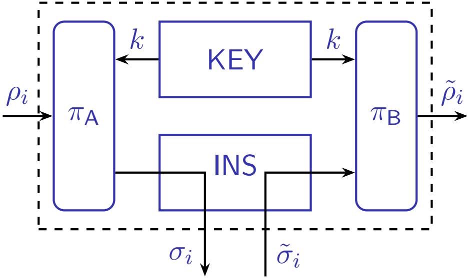

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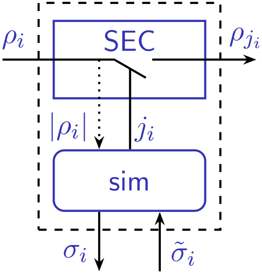

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Recall: **composable** security (= **confidentiality** + authenticity)

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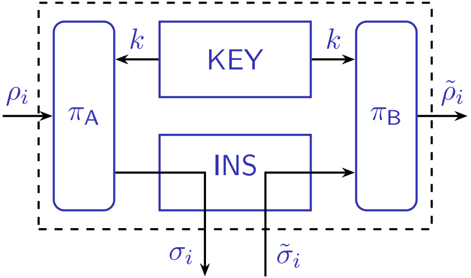

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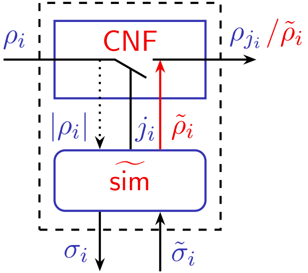

Definition

$$\boldsymbol{\pi} := (\pi_A, \pi_B)\ \varepsilon\text{-conf.} \quad :\Longleftrightarrow \quad [\text{KEY}, \text{INS}] \xrightarrow{\boldsymbol{\pi},\varepsilon} \text{CNF} \quad :\Longleftrightarrow \quad \exists\, \widetilde{\text{sim}} : \mathbf{R} \approx_\varepsilon \widetilde{\mathbf{S}}$$

Comparing QROR-CCA2 and composable confidentiality of QSE

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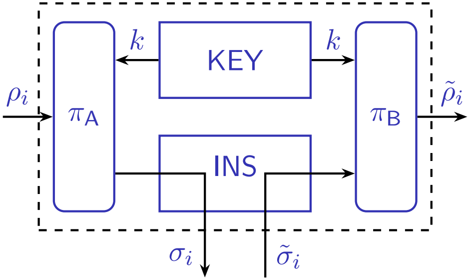
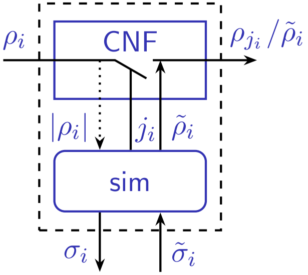
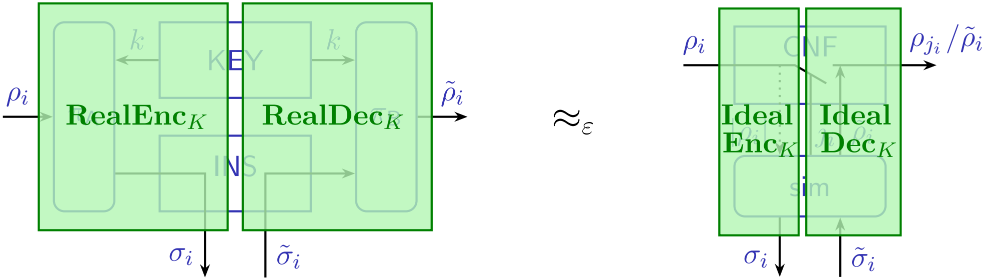


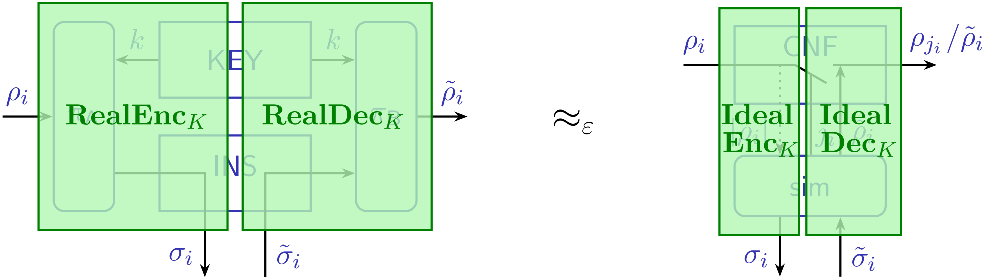
$\approx_\varepsilon$



Comparing QROR-CCA2 and composable confidentiality of QSE



Comparing QROR-CCA2 and composable confidentiality of QSE



Theorem

QCCA2 is *composable confidentiality* with a simulator hard-coded.

The End